# PHYS103 — Physics Fundamentals 3: Waves and Optics

Alianjecon Otero
alianjecon_otero@dlsu.edu.ph

July 14, 2025

# Contents

# 0 Introduction

# 1 Periodic Motion

## 1.1 Describing Oscillation

The amplitude of the motion $A$, is the maximum magnitude of displacement from the equilibrium.

The period $T$, is the time to complete one cycle. The SI unit is second.

The frequency $f$, is the number of cycles in a unit of time. The SI unit is hertz. It is the reciprocal of $T$.

$$f = \frac{1}{T}$$

$$T = \frac{1}{f}$$

The angular frequency $\omega$ is the frequency in rad/cycles. Given that a rad cycles every $2\pi$, the formula for $\omega$ is:

$$\omega = 2\pi f$$

$$\omega = \frac{2\pi}{T}$$

This means that in a $T$ amount of time $\omega$ would complete one rotation, or in one unit of time, $\omega$ completes a $f$ amount of full rotations.

## 1.2 Simple Harmonic Motion

The simplest kinds of oscillations are when the restoring force $m\ddot{x}$ is directly proportial to the displacement from the equilibrium $x$. This is also known as Hooke's law. The constant of proportionality between the restoring force and the displacement is $k$.

$$m\ddot{x} = -kx$$

Solving for the acceleration of an object in SHM, we get that:

$$\ddot{x} = -\frac{k}{m}x$$

An object that undergoes simple harmonic motion is called a harmonic oscillator. Not all periodic motions are simple harmonic, but in most systems the restoring force is approximately proportional to the displacement if the displacement is sufficiently small enough.

Using differential equations, we can solve for the equation for $x$. Let $x(0) = x_0$ and $\dot{x}(0) = v_0$.

$$\ddot{x} = -\frac{k}{m}x$$

This is a linear and homogenous differential equation with constant coefficients. We assume that $e^{rt}$ is a solution to the equation. Taking the double derivative of $e^{rt}$ results in $r^2e^{rt}$

$$r^2e^{rt} = -\frac{k}{m}e^{rt}$$

Cancelling out the common term $e^{rt}$:

$$r^2 = -\frac{k}{m}$$

$$r = \pm i\sqrt{\frac{k}{m}}$$

Now we let $\omega = \sqrt{\frac{k}{m}}$. The full solution is a linear combination of the two possible solutions:

$$x(t) = C_1e^{i\omega t} + C_2e^{-i\omega t}$$

We can use Euler's identity: $e^{ix} = \cos x + i\sin x$ to simplify this into terms with sines and cosines

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

Since $x(0) = x_0$, $A = x_0$:

$$x(t) = x_0\cos(\omega t) + B\sin(\omega t)$$

Taking the derivative:

$$\dot{x}(t) = -x_0\omega\sin(\omega t) + B\omega\cos(\omega t)$$

Since $\dot{x}(0) = v_0$, $B = \frac{v_0}{\omega}$:

$$x(t) = x_0\cos(\omega t) + \frac{v_0}{\omega}\sin(\omega t)$$

This is my preferred form of the equation. You can combine linear combinations of sine and cosine with the same coefficients into a singular term:

$$x(t) = A\cos(\omega t + \phi)$$

Where $A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$ is the maximum amplitude of the function, and $\phi$ is the phase shift of the function, where $\tan(\phi) = -\frac{v_0}{x_0\omega}$.

The total energy of the simple harmonic oscillator is given by:

$$E = \frac{1}{2}kA^2$$

Angular simple harmonic motion:

$$\omega = \sqrt{\frac{\kappa}{I}}$$

$$\theta = \Theta \cos(\omega t + \phi)$$

### 1.3 Pendulums

We can extrapolate the simple harmonic oscillator to a pendulum:

$$m\ddot{\theta} = -\frac{g}{\ell} \sin\theta$$

This does not look like a linear relationship. We can approximate a linear relationship if $\theta$ is sufficiently small enough.

$$\sin\theta = \theta$$

$$m\ddot{\theta} = -\frac{g}{\ell}\theta$$

We can then extrapolate the formula from the spring-mass system into the simple pendulum:

$$\omega = \sqrt{\frac{g}{\ell}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$$

$$T = \sqrt{\frac{\ell}{g}}2\pi$$

$$\theta(t) = A\cos(\omega t + \phi)$$

For physical pendulums:

$$\omega = \sqrt{\frac{mgd}{I}}$$

## 1.4 Damped and Forced Oscillations

Damped oscillations:

$$x = Ae^{-(b/m)t}\cos(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

Forced oscillations:

$$A = \frac{F_{\max}}{\sqrt{\left(k - m\omega_d^2\right)^2 + b^2 + w_d^2}}$$

## 1.5 Examples

Two strings with the same unstretched length but different force constants $k_1$ and $_2$ are attached to a block with mass $m$ on a level, frictionless surface. Calculate the effective force constant $k_{eff}$ in three cases: In parallel, opposite sides and in series.

An object of mass $m$ is suspended from a uniform spring with a force constant $k$, vibrates with a frequency $f_1$. When the spring is cut in half and the same objected is suspended from one of the halves, the frequency is $f_2$. What is ratio $\frac{f_1}{f_2}$?

# 2 Mechanical Waves

## 2.1 Wave Speed

The wave speed, $v$, is defined as the product of the wavelength and the frequency of the wave:

$$v = \lambda f$$

Where $\lambda$ is the wavelength. Rearranging to find lambda:

$$\lambda = \frac{v}{f}$$

$k$ stands for the wave number. It is the number of radians of phase per unit length:

$$k = \frac{2\pi}{\lambda}$$

This describes the angular speed of the phase in space. It is the direct equivalent to $\omega$ but for the spatial dimension. We can rewrite $\omega$ in terms of $k$ using the formula $v = \lambda f$:

$$v = \lambda f$$

$$v = \lambda \frac{\omega}{2\pi}$$

$$v = \frac{\omega}{k}$$

$$vk = \omega$$

## 2.2 Wave Equation

A wave function for a sinusoidal wave propagating in the $+x$-direction is:

$$y(x,t) = A\cos(kx - \omega t)$$

A function $y(x,t)$ is a wave function if it satisfies the equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

Derivation of the wave equation:

$$L = (x_1, \ldots, x_n, \partial_1 u, \ldots, \partial_n u)$$

$$\frac{\partial L}{\partial u} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{\partial L}{\partial(\partial_i u)}$$

$$L = K - V$$

$$K = \frac{1}{2}\mu\,\mathrm{d}x\left(\frac{\partial y}{\partial t}\right)^2$$

$$\Delta s = \Delta x\sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}$$

$$\mathrm{d}s \approx \mathrm{d}x\left(1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right)$$

$$V = T\,(\mathrm{d}s - \mathrm{d}x)$$

$$V = \frac{T}{2}\,\mathrm{d}x\left(\frac{\partial y}{\partial x}\right)^2$$

$$L = \frac{1}{2}\mu\,\mathrm{d}x\left(\frac{\partial y}{\partial t}\right)^2 - \frac{T}{2}\,\mathrm{d}x\left(\frac{\partial y}{\partial x}\right)^2$$

$$\mathcal{L} = \frac{L}{\mathrm{d}x}$$

$$\mathcal{L} = \frac{1}{2}\mu\left(\frac{\partial y}{\partial t}\right)^2 - \frac{T}{2}\left(\frac{\partial y}{\partial x}\right)^2$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{\partial}{\partial x}\frac{\partial \mathcal{L}}{\partial(\partial_x y)} + \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial(\partial_t y)}$$

$$0 = \mu\frac{\partial^2 y}{\partial t^2} - T\frac{\partial^2 y}{\partial x^2}$$

$$T\frac{\partial^2 y}{\partial x^2} = \mu\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}$$

$$\sqrt{\frac{T}{\mu}} = v$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

The transverse velocity of any particle in a transverse wave $v_y$ is given by the partial derivative of the wave function with respect to time:

$$v_y(x,t) = \frac{\partial y(x.t)}{\partial t}$$

The speed of a transverse wave on a string is:

$$v = \sqrt{\frac{F}{\mu}}$$

Where $F$ is the tension on the string and $\mu$ is the mass density per unit length of the string.

In general terms, $v$ is the square root of the ratio of the restoring force of the system over the inertia resisting the return.

$v$ does not depend on $v_y$.

## 2.3 Power and Intensity of Waves

The force $F_y$ of a string is given by:

$$F_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}$$

Negative sign is needed because $F_y$ restores the position to equilibrium, and so must be negative when slope is positive.

The power of that the wave does (rate of doing work) is given by:

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

This is the instanteneous rate at which energy is transferred along the string at position $x$ and time $t$. Energy is only transferred when the strong has a nonzero slope.

Using the wave function we have:

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

$$y(x,t) = A\cos(kx - \omega t)$$

$$P(x,t) = -F \cdot -kA\sin(kx - \omega t) \cdot \omega A\sin(kx - \omega t)$$

$$P(x,t) = Fk\omega A^2 \sin^2(kx - \omega t)$$

$$P(x,t) = F\frac{\omega^2}{v}A^2 \sin^2(kx - \omega t)$$

$$P(x,t) = \sqrt{\mu F}\omega^2 A^2 \sin^2(kx - \omega t)$$

As such, the maximum power that the wave equation will transmit is:

$$P_{\text{max}} = \sqrt{\mu F}\omega^2 A^2$$

And the average is simply half of that:

$$P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$$

Intensity, denoted by $I$, is the rate of energy per time per unit area. $\frac{\text{W}}{\text{m}^2}$ is the unit.

The average wave intensity $I_1$ through a sphere of radius $r_1$ and a surface area $4\pi r_1^2$

$$I_1 = \frac{P}{4\pi r_1^2}$$

This follows the inverse square law.

$$I_1 = \frac{P}{4\pi r_1^2}$$

$$I_2 = \frac{P}{4\pi r_2^2}$$

$$\frac{I_1}{I_2} = \frac{\frac{P}{4\pi r_1^2}}{\frac{P}{4\pi r_2^2}}$$

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

## 2.4 Interference, Boundary Conditions and Superposition

Principle of Superposition: When two waves overlap, the actual displacement on the string is simply the additions of the displacements of the point on the first wave and on the second wave:

$$y(x,t) = y_1(x,t) + y_2(x_t) \cdots$$

Indeed, wave functions are additive and this still satisfies the wave equation.

## 2.5 Standing Waves on a String and Fundamental Frequencies

A wave function for a standing wave on a string, fixed end at $x = 0$ is:

$$y(x,t) = \alpha \sin kx \sin \omega t$$

$\alpha$ is twice the amplitude of $A$ of either of the original travelling waves.

$$L = n\frac{\lambda}{2}$$

$$\lambda_n = \frac{2L}{n}$$

$$f_n = n\frac{v}{2L}$$

The fundamental frequency for a string fixed at both ends is:

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$$

## 2.6 Examples

A 1.80 m long uniform bar that weighs 638 N is suspended in a horizontal position by two vertical wires that are attached to the ceiling. One wire is aluminum and the other is copper. The aluminum wire is attached to the left hand of the bar, and the copper wire is attached 0.40 m to the left of the right hand end. Each waire has length 0.600m and a circular cross section with a radius of 0.280mm. What is the fundamental frequency of transverse standing waves for each wire?

# 3 Sound and Hearing

## 3.1 Sound

Sound is a longitudinal wave in a medium.

Sound waves can be described as pressure fluctuations in the medium. The pressure of a medium can be described by:

$$p(x,t) = -B\frac{\partial y(x,t)}{\partial x}$$

$$p(x,t) = BkA\sin(kx - \omega t)$$

Thus the pressure amplitude of a sound wave is:

$$p_{\mathrm{max}} = BkA$$

In general terms, $v$ is the square root of the ratio of the restoring force of the system over the inertia resisting the return. Bulk modulus is the restoring force of expansion while $\rho$ or the mass density per unit volume describes the inertia of the system.

In fluids:

Momentum $= (\rho vtA)v_y$

$$B = \frac{-\text{Pressure Change}}{\text{Fractional Volume Change}} = \frac{\Delta P}{-\frac{Av_yt}{Avt}}$$

$$\Delta p = B\frac{v_y}{v}$$

Impulse $= \Delta pAt = B\frac{v_y}{v}At$

$$B\frac{v_y}{v}At = \rho vtAv_y$$

$$v = \sqrt{\frac{B}{\rho}}$$

In ideal gases:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$R = 8.314459848\text{J}/\text{molK}$

In a solid:

$$v = \sqrt{\frac{Y}{\rho}}$$

### 3.2 Sound Intensity

$$I = \frac{1}{2}\sqrt{\rho B}\omega^2 A^2 = \frac{p_{\text{max}}^2}{2\rho v} = \frac{p_{\text{max}}^2}{2\sqrt{\rho B}}$$

$$\beta = 10\text{dB}\log\frac{I}{I_0}$$

$I_0 = 10^{-12}$

### 3.3 Pipes

For an open pipe:

$$f_n = \frac{nv}{2L}$$

For a stopped pipe:

$$f_n = \frac{nv}{4L}$$

### 3.4 Beats

Two waves interfere constructively when they are in phase and destructively when they are a half-cycle out of phase. The result wave rises and falls in intensity, forming beats.

$$f_{beat} = f_a - f_b$$

### 3.5 Doppler Effect

The doppler effect for sound is the shift in frequency when there is motion of the source of sound $S$, the listener $L$, or both:

When a source is moving away from a listener, the waves behind the source are stretched to a longer wavelength.

A listener moving toward a stationary source hears a frequency that is higher than the source frequency.

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

Positive sign if from $L$ toward $S$, negative if opposite.

When an object travels at a speed greater than the speed of sound in ari, it creates a shock wave.

$$\sin \alpha = \frac{v}{v_S}$$

### 3.6 Examples

A long spring is often used to demonstrate longitudinal waves. Show that if a spring that obeys Hooke's law has mass $m$, length $L$, and a force constant $k$, that the speed of longitudinal waves on the spring is $v = L\sqrt{\frac{k}{m}}$. Evaluate $v$ for a spring with $m = 0.250$kg, $L = 2.00$m, and $k = 1.50$ N/m.

# 4 Electromagnetic Induction

# 5 Electromagnetic Waves

Light is an electromagnetic wave.

### Maxwell's Equations

$$\oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q_{encl}}{\epsilon_0} \qquad \text{Gauss' Law} \tag{1}$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{A} = 0 \qquad \text{Gauss' Law for Magnetism} \tag{2}$$

$$\oint \vec{E} \cdot \mathrm{d}\vec{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} \qquad \text{Faraday's Law} \tag{3}$$

$$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 \left( i_c + \epsilon_0 \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} \right)_{encl} \qquad \text{Ampere's Law} \tag{4}$$

$$\tag{5}$$

These equations apply to electric and magnetic fields in a vacuum. If a material is present, the electric constant $\epsilon_0$ and the magnetic constant $\mu_0$ are replaced by the permittivity $\epsilon$ and the permeability $\mu$ of the material.

According to Maxwell's equations, an accelerating electric charge must produce electromagnetic waves.

For example, power lines carry a strong alternating current, which means that a substantial amount of charge is accelerating back and forth and generating electromagnetic waves.

These waves can produce a buzzing sound from your car radito when you drive near the lines.

### Electromagnetic Spectrum

The electromagnetic spectrum encompasses electromagnetic waves of all frequencies and wavelengths.

Visible light is the segment of the electromagnetic spectrum that we can see. It extends from the violet end (400 nanometers) to the red end (700 nanometers).

## 5.1 Plane Electromagnetic Waves and the Speeod of Light

To begin our study of electromagnetic waves, imagine that all space is divided into two regions by a plane perpendicular to the $x$-axis.

The direction of the electric field, magnetic field and the movement of the plane are all perpendicular to each other. E and B are same magnitude and direction.

$E = cB$ Electroamagnetic wave in a vacuum.

$B = \epsilon_0 \mu_0 c E$

$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$

Inserting numerical values of these constants, we obtain that

$$c = 3.00 \times 10^8 \text{ m/s}$$

The direction of propagation of an electromagnetic wave is the direction of the cross product of the elctric and magnetic fields.

$$\frac{\partial^2 E_y(x,t)}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}$$

$$\frac{1}{v^2} = \epsilon_0 \mu_0$$

Sinusodial electromagnetic plane wave propagating in the $+x$-direction:

$$\vec{E}(x,t) = \hat{\jmath} E_{\text{max}} \cos(kx - \omega t)$$

$$\vec{B}(x,t) = \hat{k} B_{\text{max}} \cos(kx - \omega t)$$

$k$ is the wavenumber and $\omega$ is the angular frequency. The unit vectors represent their directions due to the plane wave propagating in the $\hat{\imath}$ direction.

$$E_{\text{max}} = c B_{\text{max}}$$

Speed of electromagnetic waves in a dielectric:

$$v = \frac{1}{\sqrt{\epsilon \mu}} = \frac{1}{\sqrt{K K_m}} \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{c}{\sqrt{K K_m}}$$

The ratio of speed $c$ in a vacuum to the speed $v$ in the material is known as the index of refraction $n$ of the material:

$$\frac{c}{v} = n$$

In a region of empty space where $\vec{E}$ and $\vec{B}$ fields are present, the total energy density $u$ is:

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

$$u = \epsilon_0 E^2$$

Poynting vector is the vector quantity that describes the magnitude and direction of the energy flow rate of the electric and magnetic field:

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Intensity of sinusoidal electromagnetic wave in a vacuum:

$$I = S_{av} = \frac{E_{\text{max}}B_{\text{max}}}{2\mu_0} = \frac{E_{\text{max}}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{e_0}{\mu_0}}E_{\text{max}}^2 = \frac{1}{2}e_0 c E_{\text{max}}^2$$

Electromagnetic waves carry a momentum $p$, with a momenum density of

$$\frac{\mathrm{d}p}{\mathrm{d}V} = \frac{EB}{\mu_0 c^2} = \frac{S}{c^2}$$

Flow rate is:

$$\frac{1}{A}\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{S}{c}$$

Standing waves also exist for electromagnetic waves yay

# 6 The Nature and Propagation of Light

Light is both a wave and a particle this is crazy.

$$n = \frac{c}{v}$$

The laws of reflection and refraction

The incident reflected and refracted rays and the normal to the surface all lie in the same plane. This plane of incidence is perpendicualr to the plane of the boundary surface between two materials.

The angle of reflection is equal to the angle of incidence for all wave lengths and any pair of materials.

$$\theta_r = \theta_a$$

Snell's law

$$n_a \sin\theta_a = n_b \sin\theta_b$$

Wavelength of light in a material

$$\lambda = \frac{\lambda_0}{n}$$

When $n_b < n_a$, total internal reflection occurs when

$$\sin\theta_b = \frac{n_a}{n_b} \sin\theta_a$$

Critical angle happens when

$$\sin\theta = \frac{n_b}{n_a}$$

Which means that

$$1 = \sin\theta_a$$

Polarized light makes the wave makes the thingy always lie in a plane and not what it normally does

Malus's Law:

$$I = I_{\max} \cos^2\phi$$

Intensity of polarized light = maximum transmitted intensity times the square of the cosine of the angle between polarization axis of light and polarizing axis of analyzer.

Brewster's Law

$$\tan\theta = \frac{n_b}{n_a}$$

Huygens's Principle: every point on a wavefront can be considered a source of spherical wavelets and that the propagation of the wave can be determined by the sum of these secondary wavelets.

Huygens's principle states that if the position of a wave front at one instant is known, then the position of the front at a later time can be constructed by imagining the front as a source of sec ondary wavelets. Huygens's principle can be used to derive the laws of reflection and refraction.

# 7 Geometric Optics and Optical Instruments

Plane mirror $s = -s'$

Lateral magnification

$$m = \frac{y'}{y}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

For a plane mirror, image is of the same size and same distance.

The beam of incident parallel rays converges to a focal point $F$.

$f$ is the focal length it is one half radius of curvature.

$$\frac{n_a}{s} + \frac{n_b}{s'} = 0$$

Lensmaker equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Sign rules

$s > 0$ when the object is on the incoming side of surface, $s < 0$ otherwise $s' > 0$ when the image is on the outgoing side of the surface $s' < 0$ otherwise. $R > 0$ when the center of curvature is on the outgoing side of the surface, $R < 0$ otherwise $m > 0$ when the image is erect, $m < 0$ when inverted.

A camera forms a real, inverted image of the object being photographed on a light-sensitive surface. The amount of light striking this surface is controlled by the shutter speed and the aperture. The intensity of this light is inversely proportional to the square of the f-number of the lens.

f-number of a lens $= \frac{f}{D}$ D is aperture diameter

The simple magnifier creates a virtual image whose angular size $\theta'$ is larger than the angular size $\theta$ of the object itself at a distance of 25 cm, the nominal closest distance for comfortable viewing. The angular magnification $M$ of a simple magnifier is the ratio of the angular size of the virtual image to that of the object at this distance.

In a compound microscope, the objective lens forms a first image in the barrel of the instrument, and the eyepiece forms a final virtual image, often at infinity, of the first image. The telescope operates on the same principle, but the object is far away. In a reflecting telescope, the objective lens is replaced by a concave mirror, which eliminates chromatic aberrations.

$$M = \frac{\theta'}{\theta}$$

# 8 Interference

# 9 Diffraction